

Strategy for factoring polynomials:

- Step 1. **GCF**: If the polynomial has a greatest common factor other than 1, then factor out the greatest common factor.
- Step 2. **Binomials**: If the polynomial has two terms (it is a binomial), then see if it is the *difference of two squares*: $(a^2 - b^2)$.
Remember if it is the sum of two squares, it will NOT factor.
- Step 3. **Trinomials**: If the polynomial is a trinomial, then check to see if it is a perfect square trinomial which will factor into the square of a binomial: $(a + b)^2$ or $(a - b)^2$.
❖ If it is not a perfect square trinomial, use factoring **by trial and error** or the AC method.

❖ **Strategy for factoring $ax^2 + bx + c$ by grouping (AC method):**

- Form the product ac
- Find a pair of numbers whose product is ac and whose sum is b .
- Rewrite the polynomial so that the middle term (bx) is written as the sum of two terms whose coefficients are the two numbers found in step 2.
- Factor by Grouping (as in step 4)

- Step 4. **Other polynomials**: If it has more than three terms, try to factor it by grouping.
 - Group two terms together which can be factored further
 - Use the distributive property in reverse to factor out common terms
 - Write the factors as multiplication of binomials.
- Step 5. **Final check**: See if any of the factors you have written can be factored further. If you have overlooked a common factor, you can catch it here.

Remember the following properties:	
Perfect Squares:	$(a + b)^2 = a^2 + 2ab + b^2$ and $(a - b)^2 = a^2 - 2ab + b^2$
Difference of two squares:	$a^2 - b^2 = (a - b)(a + b)$
Sum of two squares:	$a^2 + b^2$ is NOT factorable

Factoring, among other benefits, helps us simplify division of polynomials such as:

$$\frac{x^2 - 4}{x - 2}$$

Instead of trying to do the long division, let's see if we can factor the numerator so we can cancel some things out:

$$\frac{x^2 - 4}{x - 2} = \frac{(x - 2)(x + 2)}{(x - 2)} = x + 2$$

Factoring Review Worksheet

Example:	Description of steps:
$2x^5 - 8x^3 =$ $2x^3(x^2 - 4) =$ $2x^3(x+2)(x-2)$	<p>Step 1: Factor out greatest common factor ($2x^3$)</p> <p>Step 2: Determine if the remaining binomial is the difference of two squares</p> <p>Step 2: It is the difference of two squares (skip steps 3-4)</p> <p>Step 5: Can it be factored further? No</p>
$3x^4 - 18x^3 + 27x^2 =$ $3x^2(x^2 - 6x + 9) =$ $3x^2(x-3)^2$	<p>Step 1: Factor out greatest common factor ($3x^2$)</p> <p>Step 2: Determine if the remaining binomial is the difference of two squares: NOT binomial.</p> <p>Step 3: Determine if the remaining trinomial is a perfect square: It seems to be $(x-3)^2$</p> <p>Step 5: Can it be factored further? No</p>
$6a^2 - 11a + 4 =$ $6a^2 - 3a - 8a + 4 =$ $(6a^2 - 3a) + (-8a + 4) =$ $3a(2a - 1) + (-4)(2a - 1) =$ $(3a - 4)(2a - 1)$	<p>Step 1: no GCF</p> <p>Step 2: Not a binomial</p> <p>Step 3: Not a perfect square; factor by AC method (or trial & error).</p> <ol style="list-style-type: none"> Find the product of ac (24). Find two numbers whose product is ac (24) and whose sum is b (-11). The two numbers are -8 and -3. Rewrite the trinomial so the middle term is the sum of the two numbers found as coefficients. <p>Step 4: Factor by grouping.</p> <p>Step 5: Cannot be factored further.</p>
$xy + 8x + 3y + 24 =$ $(xy + 8x) + (3y + 24) =$ $x(y + 8) + 3(y + 8) =$ $(x + 3)(y + 8)$	<p>Skip steps 1-3.</p> <p>Step 4: Factor by grouping</p> <ol style="list-style-type: none"> group two terms together find GCF of each group Use distributive property to "pull out" the common term. Rewrite as product of two binomials <p>Step 5: Cannot be factored further</p>
$2ab^5 + 8ab^4 + 2ab^3 =$ $2ab^3(b^2 + 4b + 1)$	<p>Step 1: Find GCF ($2ab^3$)</p> <p>Skip step 2 (not a binomial remaining)</p> <p>Step 3-4: Not a perfect square and can't be factored.</p> <p>Step 5: Cannot be factored further.</p>
$x^2 + 5x + 6 =$ $(x + 3)(x + 2)$	<p>Skip steps 1-2</p> <p>Step 3: Not a perfect square, coefficient of first term is 1, so just reverse FOIL:</p> <ol style="list-style-type: none"> First two terms are x and x Last two terms have to multiply to be 6 and sum to be 5. The two numbers are 2 and 3. Both signs need to be positive <p>Step 4: Check the OI term to make sure it's correct. It is.</p>

Factoring Review Worksheet

Factor the following polynomials using the strategy and examples above:

Polynomial:	Factored form:
$12a^2b^2 - 3ab$	
$4x^2 - 9$	
$x^2 - 16y^2$	
$x^2 - 4x + 2xy - 8y$	
$x^2 - 9x + 20$	
$9x^2 - 12x + 4$	
$8x^3 - x^2$	
$x^2 + 49$	
$16x^3 + 16x^2 + 3x$	
$x^2 - 9x + 18$	
$6x^2 + 13x + 6$	

Factoring Review Worksheet

$2x^2 + 3x - 2$	
$5x^2 - 22x - 15$	
$3x^3 + 9x^2 - 12x$	
$x^2 + 3x - 28$	
$x^2 - 8x + 16$	
$4x^2 - 7xy + 3y^2$	
$x^3 - xy + x^2 - y$	
$8x^2 - 6x - 2$	
$x^4 - 11x^3 + 24x^2$	
$6x^4y^5 - 2x^2y^3 + 14x^3y^4$	